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PLEA FOR THE RELIABILITY OF THE MULTICRITERIA DECISION MAKING MATRIX AND THE PARAMETERIZED DUAL HESITANT DIVERGENCE METRIC IN THE FUZZY SOFT SET

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Abstract

We can choose the best option from the provided criteria by using multi-criteria decision-making. Numerous scholars have used fuzzy directed divergence for multicriteria decision making extensively in recent years. The use of parameterized Hesitant Fuzzy Soft Set theory in decision-making has also been defined by some academics. In this post, we'll look into the issue of many criteria decision making in a fuzzy setting. The introduced metric is applied to an issue of decision-making. A numerical illustration of an issue involving decision-making is shown. An illustrated example of the new define approach regarding the admission preference of a student for a postgraduate programme in the science stream is used to illustrate the analysis of a fuzzy multicriteria problem. The use of the suggested fuzzy directed divergence in decision-making situations is also demonstrated.

1. Introduction

Information theory examines issues with any system that incorporates the distribution,

Key Words and Phrases : *Fuzzy Entropy, Fuzzy Divergence, Fuzzy Directed Divergence, Decision Making*

storing, retrieval, and decision-making of information. In other words, information theory investigates all issues pertaining to the thing known as a communication system. The source of messages may be a person or machine that produces the messages. The encoder transforms the messages into an object that is suitable for transmission, such as a series of binary digits (digital computer applications), the channel is a medium over which the coded message is transmitted, and the decoder converts the received output from the channel and attempts to convert the received output back into the original message to transport information. However, due to some disturbance in the system, this cannot be done consistently.

Shannon (1948) [21] is credited with identifying information theory; the measure of information theory is known as entropy w.r.t. probability distribution. Additionally, Shannon (1948) demonstrated a number of the measure's mathematical features. The measure of information associated with the two probability distributions of discrete random variables, $p = (p_1, p_2, \cdots, p_n)$ and $q = (q_1, q_2, \cdots, q_n)$ was quantified by Kullback and Liebler [13] in 1952. This phenomenon is referred to as directed divergence, $D(pllq) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i}$. Other measures of divergence on a set of probabilities exist and go by different names, including distance and discrimination, for example. Divergence is a non-negative function that turns into zero when two sets coincide, and these are the inherent features of directed divergence.

Other measures of divergence on a set of probabilities exist and go by different names, including distance and discrimination, for example. Divergence is a non-negative function and turns into zero when two sets coincide. These are the inherent features of directed divergence.

Zadeh (1965) [35] introduced fuzzy set theory, which is similar to probability theory. Human thought is characterised by uncertainty and fuzziness, which are connected to many real-world issues. Our language, our judgement, and the path of our acts are all fuzzy. Set theory, which determines whether an element belongs to a set or not, gained a completely new dimension with the development of fuzzy set theory. A fuzzy set $\tilde{A}$ is subset of universe of discourse X, is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)|x \in X\}$ where $\mu_{\tilde{A}} : X \to [0, 1]$ is a membership function of $\tilde{A}$. The value of $\mu_{\tilde{A}}(x)$ describes the degree of belongingness of $x \in X$ in $\tilde{A}$.

Shannon entropy works with probabilistic uncertainty, whereas fuzzy entropy deals with vagueness and ambiguity. De Luca and Termini (1972) [6] described the characteristics of fuzzy entropy and listed a number of properties (1-4) that fuzzy entropy should satisfy:

1. If the set is crisp, fuzzy entropy is at its lowest.
2. When the membership value is 0.5, the fuzzy entropy is at its highest.
3. If the set is sharpened, fuzzy entropy decreases.
4. A set's fuzzy entropy is equal to its complement

Fuzzy divergence, one of several measures of fuzzy entropy and measure of fuzzy divergence that correspond to a fuzzy set $\tilde{A}$. relative to some other fuzzy set $\tilde{B}$ was introduced by Bhandari and Pal in 1993. It is a fuzzy information measure for discrimination of a fuzzy set $\tilde{A}$ relative to some other fuzzy set $\tilde{B}$. Let $F(X)$ be the collection of all fuzzy subsets of X and let X be a universal set. If a mapping $D : F(X) \times F(X) \to R$ satisfies any of the following conditions, it is said to represent a divergence between two fuzzy subsets: if it satisfies following properties for any $\tilde{A}.\tilde{B} \quad C \in \tilde{F}(X)$.

1. $D(\tilde{A}, \tilde{B})$ is non-negative.
2. $D(\tilde{A}, \tilde{B}) = D(\tilde{B}, \tilde{A})$
3. $D(\tilde{A}, \tilde{B}) = 0$ if $\tilde{A} = \tilde{B}$.
4. $Max\{\{D(\tilde{A} \cup \tilde{B} \cup \tilde{C}), D(\tilde{A} \cap \tilde{C}, \tilde{B} \cap \tilde{C})\} \leq D(\tilde{A}, \tilde{B})$.

The simplest fuzzy directed divergence is

$$D(\tilde{A}, \tilde{B}) = \sum_{i=1}^{n} \left[\mu_A(x_i) \log \frac{\mu_A(x_i)}{\mu_B(x_i)} + (1 - \mu_A(x_i)) \log \frac{(1 - \mu_A(x_i))}{(1 - \mu_B(x_i))} \right]$$

given by Bhandari and Pal [1] (1993), where $\mu_A(x_1), \mu_A(x_2), \cdots, \mu(A(x_n)$ describes the degree of belongingness of $x_i \in X$ in A and $\mu_B(x_1), \mu_B(x_2), \cdots, \mu_B(x_n)$ describes the degree of belongingness of $x_i \in X$ in B respectively. Later, Fan and Xie (1999) gave the discrimination of fuzzy information of fuzzy set A against B.

$$I(A, B) = \sum_{i=1}^{n} \left[1 - (1 - \mu_A(x_i)) e^{\mu_A(x_i) - \mu_B(x_i)} - \mu_A(x_i) e^{(\mu_B(x_i) - \mu_A(x_i))} \right]$$

with respect to exponential fuzzy entropy given by Pal and Pal[1] (1989). Further, corresponding to entropy given by Havrda-Charvat [9] (1967), Kapur [11](1997) gave a generalized measure of fuzzy directed divergence as

$$\begin{aligned} I_\alpha(A,B) &= \frac{1}{\alpha-1}\sum_{i=1}^{n}\left[\mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i)+(1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha}-1\right] \\ &\qquad \alpha\neq 1,\quad \alpha>0. \end{aligned}$$

Divergence is found in various applications in the real world such as image segmentation, medical sciences, pattern recognition, fuzzy clustering etc.
These operations are used to combine two or more fuzzy sets into one. An aggregation operation (Klir & Folger, 1988)[12] defined as a function $A:[0,1]^n\to[0,1]$ satisfying:

1. $A(0,0,\cdots,0)$ and $A(1,1,\cdots,1)=1$.

2. A is monotonic in each argument.

Bhatia and Singh [2] (2013), proposes a measure of arithmetic-geometric directed divergence of two arbitrary fuzzy sets A and B is as

$$\begin{aligned} T(A,B) &= \sum_{i=1}^{n}\left[\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\log\frac{\mu_A(x_i)+\mu_B(x_i)}{2\sqrt{\mu_A(x_i)+\mu_B(x_i)}}\right. \\ &\left.+\frac{2-m_A(x_i)-\mu_B(x_i)}{2}\log\frac{2-\mu_A(x_i)-\mu_B(x_i)}{2\sqrt{2}-\mu_A(x_i)-\mu_B(x_i)}\right] \end{aligned}$$

and defined generalized triangular discrimination between two arbitrary fuzzy sets A and B as follows:

$$\begin{aligned} \Delta_\alpha(A,B) &= \sum_{i=1}^{n}(\mu_A(x_i)-\mu_B(x_i))^{2\alpha}\left[\frac{1}{(\mu_A(x_i)+\mu_B(x_i))^{2\alpha-1}}\right. \\ &\left.+\frac{1}{(2-\ _A(x_i)-\mu_B(x_i))^{2\alpha-1}}\right]. \end{aligned}$$

They also defined a new class of measure of fuzzy directed divergence for two arbitrary sets A and B as

$$\begin{aligned} D_\alpha^\beta(A,B) &= \frac{1}{\beta-1}\sum_{i=1}^{n}\left[\left(\mu_A(x_i)^\alpha\mu_B(x_i)^{1-\beta}+(1-\mu_A(x_i))^{1-\alpha}(1-\mu_B(x_i))^{1-\alpha}\right)^{\frac{\beta-1}{\alpha-1}}-1\right] \\ &\qquad \alpha>0,\quad \alpha\neq 1,\quad \beta>0,\quad \beta\neq 1. \end{aligned}$$

and introduced (α, β) generalized arithmetic-geometric measure of fuzzy directed divergence

$$T_\alpha^\beta(A,B) = \frac{1}{2}\left[D_\alpha^\beta\left(\frac{A+B}{2}, A\right) + D_\alpha^\beta\left(\frac{A+B}{2}, B\right)\right]$$

and $T_\alpha^\beta(A,B) = T(A,B)$ at $\alpha = \beta = 1$.

A new measure of fuzzy directed divergence for two Fuzzy sets A and B,

$$M_{H^*\ A^*}^F(A,B) = \sum_{i=1}^{n} \frac{(\mu_A(x_i) - \mu_B(x_i))^2}{2}\left[\frac{1}{\mu_A(x_i) - \mu_B(x_i)} + \frac{1}{2 - \mu_A(x_i) - \mu_B(x_i)}\right]$$

where $A^* : [0,1]^2 \to [0,1]$ such that $A^*(a,b) = \frac{a+b}{2}$ and $H^* : [0,1]^2 \to [0,1]$ such that $H^*(a,b) = \frac{a^2+b^2}{a+b}$ was defined by Bhatia and Singh (2013), they also discussed application of new directed divergence measure in images segmentation.

Bhatia and Singh [4] (2013), introduced three new divergence measures between fuzzy sets and some properties of these divergence measures. They all defined three aggregation functions corresponding to divergence measures. Verma [24, 26, 27, 28, 29] et al. (2012),defined a measure of entropy as

$$V_a(P) = \sum_{i=1}^{n} ln(1 + ap_i) - \sum_{i=1}^{n} ln\ p_i - ln(1 + a), \quad a > 0$$

for probability distribution, and its corresponding measure of directed divergence is defined as

$$D_\alpha(P:Q) = \sum_{i=1}^{n} q_i ln\frac{pi}{q_i} - \sum_{i=1}^{n} q_i ln\left(\frac{q_i + ap_i}{q_i}\right) + ln(1+a), \quad a > 0$$

and corresponding measure of fuzzy directed divergence is

$$\begin{aligned} D(A,B) &= \sum_{i=1}^{n} \mu_B(x_i) ln\left(\frac{m_A(x_i)}{a\mu_A(x_i) + \mu_B(x_i)}\right) \\ &+ \sum_{i=1}^{n} (1 - \mu_B(x_i)) ln\left(\frac{1 - \mu_A(x_i)}{1 + a - a\mu_A(x_i) - \mu_B(x_i)}\right) + ln(1+a), \quad a > 0 \end{aligned}$$

and their properties were studied.

Bhagwandas [5] et al. (2021) proposed a new fuzzy directed divergence as follows

$$\begin{aligned} H_{\alpha,\beta}(A,B) &= \frac{1}{(\alpha-1)\beta} \sum_{i=1}^{n} \left\{\left[\mu_A^\alpha(x_i)\mu_b^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(xi))^{1-\alpha}\right]^\beta - 1\right\} \\ & \alpha 0, \quad \alpha \neq 1, \quad \beta \neq 0. \end{aligned}$$

Verma [32, 33, 34] gave a measure of fuzzy directed divergence is

$$H_{\alpha,b}(A,B) = \frac{1}{(\alpha-1)\beta}\sum_{i=1}^{n}\log\left\{\left[\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}(1-\mu_B(x_i))^{1-\alpha}\right]-1\right\}$$
$$\alpha>0,\quad \alpha\neq 1,\quad \beta\neq 0.$$

Li et al. (2014a) [14] suggested two methods based on fuzzy equivalences and dissimilarity functions to define divergence measure in fuzzy environments. This type of method has also been discussed by Verma [25, 30, 31]. Different approaches to defining fuzzy equivalences were put out by Li et al. (2014b)[15], who then utilised them to develop fuzzy set similarity measures. Inequalities between several fuzzy mean divergence measures are presented by Tomar and Ohlan (2014a). Tomar and Ohlan (2014b) [22] establish a connection between the suggested divergence measure and the entropy of order by introducing a parametric generalised exponential measure of fuzzy divergence of order.

A generalised measure of fuzzy divergence was put out by Tomar and Ohlan (2015) [23], who used it to solve multi-criteria decision-making issues. The robustness of fuzzy connectives and reasoning utilising generic divergence measure was studied by Li et al. in 2016[16]. He et al. (2016) demonstrated that the similarity measures established by Li et al. (2014) satisfy the T transitivity and also looked into its fuzzy equivalents. We have proposed two new binary aggregation operations in the following section, and a new divergence measure has been developed to match these operators.

2. Our Results

1. Application

Multicriteria decision making is a very useful and practical technique in the real world. We can choose the best option among the available options by applying multi-criteria decision-making. Numerous scholars have used fuzzy directed divergence for multicriteria decision making extensively in recent years. In this post, we'll look into the issue of many criteria decision making in a fuzzy setting. We have a collection of strategies for multi-criteria decision-making, such as $A_1, A_2, \cdots, A_n$. Assume that each strategy's level of efficacy in relation to the budget is different $C_1, C_2, \cdots, C_m$.

Step 1. First we arrange the preference of decision makers in the form of fuzzy decisionmaking matrix for each alternative $A_J (j = 1, 2, \cdots, n)$ w.r.t. cost set C_k $(k =$

$1, 2, \cdots, m)$ as follows

$$D_{m\times n}[a_i] = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}_{n\times m} \end{array}$$

Step 2. We determine the ideal solution from all alternative corresponding to their cost set as

$$A^* = \{H_1^*, H_2^*, \cdots, H_n^*\} \quad \text{where} \quad H_l^* = \max\{H_i^*\}.$$

Step 3. Therefore we calculate the divergence using $H_{\alpha,\beta}(A : B)$ given as

$$\begin{aligned} H_{\alpha,\beta}(A, B) \quad &= \quad \frac{1}{(\alpha-1)\beta} \sum_{i=1}^{n} \left\{ \left[\mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \right]^\beta - 1 \right\} \\ & \qquad \alpha > 0, \quad \alpha \neq 1, \quad \beta \neq 0. \end{aligned}$$

Step 4. To give ranking we take

$$\min\{H_{\alpha,\beta}(A_j, A^*)\}; \quad \text{where} \quad 1 \leq j \leq n.$$

Select the most desirable alternative according with descending order of their function.

Numerical Example

In a fuzzy multicriteria problem we analyze an illustration example of the new define approach regarding admission preference of a student for Ph. D. course of science stream. Suppose that the students wants to take admission in university and he want to select an institute from five option

A_1 = Bharti University Durg

A_2 = Guru Ghasidas University Bilaspur

A_3 = Pt. Ravishankar Shukla University Raipur

A_4 = Bastar University Jagdalpur

A_5 = Dr. C.V. Raman University Kota.

These are the most valuable institute for science course.The Student wants to choose university on the following basis

C_1 = Placement

C_2 = Ranking

C_3 = Faculty

C_4 =Facility

C_5 = Fee

Step 1. Arranging the value of all alternative we arrange the preference in matrix $M_{n\times m}[m_{ij}]$

$$M_{n\times m}[m_{ij}] = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0.5 & 0.7 & 0.8 & 0.3 & 0.1 \\ 0.7 & 0.9 & 0.3 & 0.2 & 0.4 \\ 0.4 & 0.8 & 0.6 & 0.7 & 0.5 \\ 0.3 & 0.1 & 0.5 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.8 & 0.6 \end{bmatrix} \end{array}$$

Step 2. Optimum solution from above matrix is

$$A^* = \{0.7, 0.9, 0.8, 0.8, 0.6\}.$$

Step 3. We calculate the divergence of A^* w.r.t. each alternative as

Table 1

α	β	$H_{\alpha,\beta}(A_1, A^*)$	$H_{\alpha,\beta}(A_2, A^*)$	$H_{\alpha,\beta}(A_3, A^*)$	$H_{\alpha,\beta}(A_4, A^*)$	$H_{\alpha,\beta}(A_5, A^*)$
0.1	0.1	0.14	0.14	0.03	0.31	0.25
0.9	0.1	1.25	1.36	0.34	2.81	2.33
1.1	2.1	1.56	1.74	0.43	3.68	3.05
5	2.1	30.25	72.54	2.60	1960.1	960.5

Step 4. From the table 1 we find that $H_{\alpha,\beta}(A_3, A^*)$ has minimum value for all parameters so we can easily estimate that the best alternative is . So the student should take admission in Pt. Ravishankar University Raipur.

2. Application

In the actual world, multicriteria decision making is a very beneficial and useful strategy. By using multi-criteria decision-making, we may select the best alternative from those that are offered. In recent years, fuzzy directed divergence has been widely employed for multicriteria decision making by a number of academics. We'll examine the problem of making decisions with multiple criteria in a fuzzy environment in this topic. We have a variety of methods for making decisions based on several criteria, including

$A_1, A_2, \cdots, A_n$. Assume that the effectiveness of each method in proportion to the budget varies $C_1, C_2, \cdots, C_m$.

Step 1. First we arrange the preference of decision makers in the form of fuzzy decision making matrix for each alternative

$$A_J \ (j = 1, 2, \cdots, n) \ \ w.r.t. \ \ cost\ set \ \ C_k \ (k = 1, 2, \cdots, m)$$

as follows

$$D_{m\times n}[a_{ij}] = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}_{n\times m} \end{array}$$

Step 2. We determine the ideal solution from all alternative corresponding to their cost set as

$$A^* = \{H_1^*, H_2^*, \cdots, H_n^*\} \ \text{ where } \ H_l^* = \max\{H_i^*\}.$$

Step 3. Therefore we calculate the divergence using $H_{\alpha,\beta}(A : B)$ given as

$$H_{\alpha,\beta}(A, B) = \frac{1}{(\alpha-1)\beta} \sum_{i=1}^{n} \log\left\{ \left[\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}(1-\mu_B(x_i))^{1-\alpha}\right]^{\beta} - 1 \right\}$$
$$\alpha > 0, \quad \alpha \neq 1, \quad \beta \neq 0.$$

Step 4. To give ranking we take

$$\min\{H_{\alpha,\beta}(A_j, A^*)\}; \quad \text{where } \ 1 \leq j \leq n.$$

Select the most desirable alternative according with descending order of their function.

Numerical Example

In a fuzzy multicriteria problem we analyze an illustration example of the new define approach regarding preference of a customer for Two Wheeler. Suppose that the customer wants to buying two wheeler and he want to select a company from five option

A_1 = Hero

A_2 = Honda

A_3 = TVS

A_4 = Bajaj

A_5 = Suzuki.

These are the most valuable company for two wheeler.

The Customer wants to choose company on the following basis

C_1 =Mileage

C_2 =Availability

C_3 =Weight

C_4 =Insurance Policy

C_5 = Price

Step 1. Arranging the value of all alternative we arrange the preference in matrix

$$M_{n\times m}[m_{ij}] = \begin{array}{c c} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{bmatrix} 0.5 & 0.7 & 0.8 & 0.3 & 0.1 \\ 0.7 & 0.9 & 0.3 & 0.2 & 0.4 \\ 0.4 & 0.8 & 0.6 & 0.7 & 0.5 \\ 0.3 & 0.1 & 0.5 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.8 & 0.6 \end{bmatrix} \end{array}$$

Step 2. Optimum solution from above matrix is

$$A^* = \{0.7, 0.9, 0.8, 0.8, 0.6\}.$$

Step 3. We calculate the divergence of A^* w.r.t. each alternative as

Table 2

α	β	$H_{\alpha,\beta}(A_1, A^*)$	$H_{\alpha,\beta}(A_2, A^*)$	$H_{\alpha,\beta}(A_3, A^*)$	$H_{\alpha,\beta}(A_4, A^*)$	$H_{\alpha,\beta}(A_5, A^*)$
0.1	0.1	-0.85	-0.85	-1.52	-0.50	-0.60
0.9	0.1	0.09	0.133	-0.46	0.44	0.36
1.1	2.1	0.19	0.24	-0.36	0.56	0.48
5	2.1	1.48	1.86	0.41	3.29	2.98

Step 4. From the table 1 we find that $H_{\alpha,\beta}(A_3, A^*)$has minimum value for all parameters so we can easily estimate that the best alternative is A_3. So the Customer should buy two wheeler from TVS company.

3. Application

In this section, we present a method to solve decision-making problems using proposed fuzzy directed divergence.

$$\begin{aligned} F_{G,K}(A,B) &= \sum_{i=1}^{n}\Bigg[\frac{(\mu_A(x_i)+\mu_B(x_i))^4}{8(\mu_A(x_i)^2+\mu_B(x_i)^2)} - \mu_A(x_i)\mu_B(x_i) \\ &\quad + \frac{(2-\mu_A(x_i)-\mu_B(x_i))^4}{8((1-\mu_A(x_i))^2+(1-\mu_B(x_i))^2)} - (1-\mu_A(x_i))(1-\mu_B(x_i))\Bigg]. \end{aligned}$$

Making decisions means considering a variety of options in an unknowable world. The decision maker must choose the best plan of action from the available options before making a choice. Diverse study designed numerous divergence, similarity, and entropy measurements to choose the optimum course of action. Let's imagine a decision-making issue involving a collection of possibilities based on the divergence measure. $P = \{P_1, P_2, \cdots, P_m\}$ to be taken into account based on specific criterion $D = \{C_1, C_2, \cdots, C_n\}$. The optimum solution P_* to the problem is having the highest membership values possible for each criterion, and characteristic sets for each choice are determined by assigning suitable values to membership values. Each case's divergence is calculated, and the option with the least divergence is chosen.

We take into account a few decision-making issues to demonstrate the usefulness of the suggested fuzzy directed divergence.

Numerical Example

Suppose customers want to buy a smart watch. Customer wants to select a service provider from five options: A_1, A_2, A_3, A_4, A_5 smart watch providers on the basis of

P_1 =Sleep Pattern Monitoring

P_2 = Step Counting

P_3 = Blood Pressure Monitoring

P_4 =Heart Rate Monitoring.

For evaluating five alternatives, the decision makers form five fuzzy sets as

$$\begin{aligned} A_1 &= \{(P_1, 0.5), (P_2, 0.6), (P_3, 0.3), (P_4, -.2)\} \\ A_2 &= \{(P_1, 0.7), (P_2, 0.7), (P_3, 0.7), (P_4, 0.4)\} \\ A_3 &= \{(P_1, 0.6), (P_2, 0.5), (P_3, 0.5), (P_4, 0.6)\} \\ A_4 &= \{(P_1, 0.8), (P_2, 0.6), (P_3, 0.3), (P_4, 0.2)\} \\ A_5 &= \{(P_1, 0.6), (P_2, 0.4), (P_3, 0.7), (P_4, 0.5)\}. \end{aligned}$$

Optimal Solution is

$$A_* = \{(P_1, 0.8), (P_2, 0.7), (P_3, 0.7), (P_4, 0.6)\}.$$

Divergence of A_*, w.r.t. each option given as

$$\begin{aligned} D(A_1, A_*) &= 0.027728 \\ D(A_2, A_*) &= 0.000876 \\ D(A_3, A_*) &= 0.002917 \\ D(A_4, A_*) &= 0.023099 \\ D(A_5, A_*) &= 0.005059. \end{aligned}$$

Optimal solution is with minimum divergence is A_2 with preference order given as A_2, A_3, A_5, A_4, A_1. So customer should buy smart watch from operator A_2.

4. Application

In this section, we present a method to solve decision-making problems using proposed fuzzy directed divergence.

$$\begin{aligned} F_{G,K}(A,B) &= \sum_{i=1}^{n} \log \left[\frac{(\mu_A(x_i) + \mu_B(x_i))^4}{8(\mu_A(x_i)^2 + \mu_B(x_i)^2)} - \mu_A(x_i)\mu_B(x_i) \right. \\ &\quad \left. + \frac{(2 - \mu_A(x_i) - \mu_B(x_i))^4}{8((1-\mu_A(x_i))^2 + (1-\mu_B(x_i))^2)} - (1-\mu_A(x_i))(1-\mu_B(x_i)) \right]. \end{aligned}$$

Making choices in an uncertain environment requires weighing a range of options. Before making a decision, the decision-maker must select the best course of action among the available possibilities. To select the best course of action, many studies created numerous divergence, similarity, and entropy metrics. Consider a situation where you have to choose between a range of options depending on the divergence metric. Taking into account $P = \{P_1, P_2, \cdots, P_m\}$ based on a specified requirement $D = \{C_1, C_2, \cdots, C_n\}$. The problem can be solved best by having the highest membership values for each criterion; characteristic sets for each option are determined by giving membership values appropriate values. The option with the least amount of divergence is selected after the divergence for each case is determined.

To show the value of the recommended fuzzy directed divergence, we take into account a few decision-making challenges.

Numerical Example

A wants to open manufacturing bricks of cement and they need to select the location out of six locations $L_1, L_2, L_3, L_4, L_5, L_6$ on the basis of

$D_1 =$ Business Climate

$D_2 =$ Quality of Labour

$D_3 =$ Quality of Labour

$D_4 =$ Suppliers

$D_5 =$ Total Costs

$D_6 =$ Proximity of customers

$D_7 =$ Free Trade Zone.

For evaluating six locations, the management formed six fuzzy sets as follows:

	D_1	D_2	D_3	D_4	D_5	D_6	D_7
L_1	0.,4	0.7	0.5	0.9	0.4	0.6	0.6
L_2	0.7	0.9	0.6	0.7	0.6	0.6	0.8
L_3	0.9	0.6	0.4	0.5	0.7	0.5	0.3
L_4	0.5	0.5	0.6	0.3	0.6	0.8	0.7
L_5	0.6	0.5	0.7	0.6	0.7	0.5	0.5
L_6	0.4	0.3	0.2	0.5	0.5	0.4	0.3

Optimal solution is

$$L_* = \{(D_1, 0.9), (D_2, 0.9), (D_3, 0.7), (D_4, 0.9), (D_5, 0.7), (D_6, 0.8), (D_7, 0.8)\}.$$

Divergence of from each given option $L_1, L_2, L_3, L_4, L_5, L_6$ is given as

$$\begin{aligned} D(L_1, L_*) &= -1.4157 \\ D(L_2, L_*) &= -2.249 \\ D(L_4, L_*) &= -1.089 \\ D(L_5, L_*) &= -1.417 \\ D(L_6, L_*) &= -0.799. \end{aligned}$$

The optimal solution is with the minimum divergence. So management should open factory of cement bricks at location L_2, with preference order $L_2, L_5, L_1, L_3, L_4, L_6$.

Conclusion

In this article we describe fuzzy directed divergence measure is very suitable measure to solve the multicriteria decision making problem. Some fuzzy directed divergence measure has been applied to a few illustrative examples of decision making problems, which shows how it helps in decision making by minimizing fuzzy directed divergence.

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